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Report by the Astronomer Royal on the Present State of the Calculations in his New Lunar Theory.

In two preceding communications to the Society, which are printed in their *Monthly Notices*, 1874 January, and 1875 March, I have explained as the principles of the method proposed by me, that, in the plane of the ecliptic, there are formed the equations of areas and of *vis viva*, and, normally to the ecliptic, the simple equation of the normal forces; that these equations are used without integration; and that the peculiarity of the method consists in the following process:—that Delaunay's numerical values for every term in the detailed expansion of Parallax, Longitude, and Latitude, are adopted, every term being supplemented by addition of symbolical corrections both to the coefficient and to the argument (different for every different term); and that in this state the terms are substituted in the equations to which I have alluded; and that thus subordinate equations will be formed by which the numerical value of every symbolical correction may be obtained.

The left side of the equations, as exhibited in the paper of 1874 January, consists entirely of functions of the co-ordinates as disturbed, without explicit introduction of the disturbing forces. In these (omitting mention of the terms depending on the earth's oblateness and the terms depending on the change of plane of the ecliptic, in which no important work is yet done), the following steps are made:—

The whole of the substitutions are made to the decimal figure 10^{-7}. And these have been done with so much care, the whole in duplicate and portions in triplicate, and all subjected to irregular examination, that I would fain hope that there are no systematic errors, and very few and unimportant special errors.

There are, however, terms, whose treatment, though generally in the same form, requires distinct attention.

First, there are the terms of long period. These terms have small factors in $\frac{dv}{dt}$; and in the differentiation of every function of the ordinates, and if no special care were taken, their results would be lost in the equations. A list has been prepared, therefore, of about twenty terms in longitude, and about as many in latitude, in which the "movement of argument" is small. (The first in longitude is the Annual Equation; the last is the distance of the lunar perigee from the solar perigee. The first in latitude is the distance of lunar node from lunar perigee; and the last is the distance of lunar node from solar perigee.) The risk of error here is generally removed by adopting the rule,

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that the values of $\frac{d v}{d t}$ and the like are to be taken so far in the decimal scale, that the number of effective figures in the product shall be as great as that in the multiplicand. Some of these products, repeated, will advance to 10^{-11}, or further.

Secondly, there are the terms whose movements of arguments differ little from the movement of Moon's longitude. These terms are, in general, closely related to those first mentioned. They require the same attention as the first, not because the factors in $\frac{d v}{d t}$, &c., are small, but because the factors of symbolical correction, on the right side and on the left side, approach to equality. They never require to be carried so far as those of the first series. The principal of these terms are the Evections in longitude and in latitude.

Some progress, but not great, is made in this supplementary work. It will require much of my personal attention.

Finally, for the left side of the equations, there are to be introduced the coefficients of the symbolical corrections. A considerable advance has been made in these, as applying to coefficients of angular or periodic terms; but there is yet much to be done. As applying to arguments, no part is yet done. This work of symbolical corrections will also require my own attention.

I proceed now to report on the progress in the preparation of the right side of the equations.

Different terms in the right side are multiplied by different powers of the fraction $\frac{\text{Moon's distance}}{\text{Sun's distance}}$, whose value differs little from $\frac{1}{180}$. If, then, our principal term is computed to the decimal 10^{-7}, it will suffice to compute the next to 10^{-5}, the following term to 10^{-3}, and so on. The calculation of these different powers is completed, requiring my examination.

Each of these powers multiplies a function (some of them rather complicated) of sines and cosines, of the difference between Sun's true longitude and Moon's true longitude, and of sines and cosines of multiples of that difference. If the Moon's true longitude = Moon's mean longitude + ζ, and if the Sun's true longitude = Sun's mean longitude + θ, and if $\zeta - \theta = \chi$, then the difference of the true longitudes = difference of mean longitudes + χ. In order to prepare the sines and cosines above mentioned, it is necessary to prepare the successive powers of χ to the sixth; by means of these, the several series representing the sines and cosines of the multiples of χ may be formed; and, combining these (by the usual trigonometrical formulæ) with the sines and cosines of multiples of difference of mean longitude, the sines and cosines in question will be

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formed. The powers of χ are prepared, and advances are made in the other steps, all requiring my examination.

The principal risk of error in these operations, conducted by young men of small mathematical education, is in the formation of the products of sines by cosines, when the arguments of both factors are somewhat complicated.

Royal Observatory, Greenwich,
1875, *December* 17.

Addendum to the preceding Report.

I have remarked above that there are, in the Lunar Theory, certain coefficients whose values must be computed to a greater degree of accuracy than is necessary for the general mass of calculations; the selection of the terms depending on one of these circumstances: either that the inequalities which they represent go through their periodic changes in the course of many revolutions of the Moon, or that they go through their changes in a period differing slightly from a single revolution of the Moon. For the sake of placing on record more accurately the state of my own progress in the Theory, as well as for providing numbers which ought, in my opinion, to be used in their numerical form (as distinguished from a symbolical form) in all Lunar Theories, I ask leave to present to the Society, Tables of the numerical values of the Movements of Arguments to which I allude.

The unit of time which I adopt is that which in the usual expression of Moon's mean longitude in terms of radius makes $nt=t$, or which equals the Moon's periodic time divided by 2π. Then the following may be adopted as fundamental numbers; which theoretically undergo infinitesimal changes, but which for our purposes may be regarded as permanent constants; subject to corrections in the values for f and l, which it is one object of the Theory to ascertain :

D = mean angular departure of the Moon from the Sun = +0·9251987
f = mean angular departure of the Moon from Ascending Node = +1·0040219
l = mean angular departure of the Moon from Lunar Perigee = +0·9915480
L = mean angular departure of the Sun from Solar Perigee = +0·0748006

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(A). The following Movements apply to inequalities of angular motion parallel to the plane of the Ecliptic.

1. *Inequalities of Long Period.*

Arguments.				Movements.
$+D$	$-2f$	$+l$		$-0{\cdot}0912971$
$+D$	$-2f$	$+l$	$+L$	$-0{\cdot}0164965$
$+D$		$-l$		$-0{\cdot}0663493$
$+D$		$-l$	$+L$	$+0{\cdot}0084513$
$+D$		$-l$	$-L$	$-0{\cdot}1411499$
$+2D$	$+2f$	$-4l$		$-0{\cdot}1077508$
$+2D$	$-2f$			$-0{\cdot}1576464$
$+2D$	$-2f$		$+L$	$-0{\cdot}0828458$
$+2D$	$-2f$		$-L$	$-0{\cdot}2324470$
$+2D$	$-2f$		$+2L$	$-0{\cdot}0080452$
$+2D$		$-2l$		$-0{\cdot}1326986$
$+2D$		$-2l$	$+L$	$-0{\cdot}0578980$
$+2D$		$-2l$	$-L$	$-0{\cdot}2074992$
$+2D$		$-2l$	$+2L$	$+0{\cdot}0169026$
$+2D$		$-2l$	$-2L$	$-0{\cdot}2822998$
$+3D$	$-2f$	$-l$		$-0{\cdot}2239957$
$+3D$		$-3l$		$-0{\cdot}1990479$
	$+2f$	$-2l$		$+0{\cdot}0249478$
	$+2f$	$-2l$	$+L$	$+0{\cdot}0997484$
			$+L$	$+0{\cdot}0748006$
			$+2L$	$+0{\cdot}1496012$
			$+3L$	$+0{\cdot}2244018$

2. *Inequalities whose Periods differ little from the Time of the Moon's Revolution.*

Arguments.				Movements.
$+D$				$+0{\cdot}9251987$
$+D$	$+2f$	$-2l$		$+0{\cdot}9501465$
$+D$	$-2f$			$-1{\cdot}0828451$
$+D$	$-2f$	$+2l$		$+0{\cdot}9002509$
$+D$	$-2f$		$+L$	$-1{\cdot}0080445$
$+D$		$-2l$		$-1{\cdot}0578973$
$+D$		$-2l$	$+L$	$-0{\cdot}9830967$

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Arguments.				Movements.
$+D$			$-L$	$+0{\cdot}8503981$
$+D$			$+2L$	$+1{\cdot}0747999$
$+D$			$-2L$	$+0{\cdot}7755975$
$+2D$	$+2f$	$-3l$		$+0{\cdot}8837972$
$+2D$	$-2f$	$+l$		$+0{\cdot}8339016$
$+2D$	$-2f$	$+l$	$+L$	$+0{\cdot}9087022$
$+2D$	$-2f$	$+l$	$-L$	$+0{\cdot}7591010$
$+2D$	$-2f$	$-l$		$-1{\cdot}1491944$
$+2D$	$-2f$	$-l$	$+L$	$-1{\cdot}0743938$
$+2D$	$-2f$	$-l$	$-L$	$-1{\cdot}2239950$
$+2D$		$-l$		$+0{\cdot}8588494$
$+2D$		$-l$	$+L$	$+0{\cdot}9336500$
$+2D$		$-l$	$+2L$	$+1{\cdot}0084506$
$+2D$		$-l$	$-L$	$+0{\cdot}7840488$
$+2D$		$-3l$		$-1{\cdot}1242466$
$+2D$		$-3l$	$+L$	$-1{\cdot}0494460$
$+2D$		$-3l$	$+2L$	$-0{\cdot}9746454$
$+2D$		$-3l$	$-L$	$-1{\cdot}1990472$
$+2D$		$-3l$	$-2L$	$-1{\cdot}2738478$
$+3D$	$-2f$			$+0{\cdot}7675523$
$+3D$		$-2l$		$+0{\cdot}7925001$
$+3D$		$-2l$	$+L$	$+0{\cdot}8673007$
$+4D$		$-3l$		$+0{\cdot}7261508$
$+4D$		$-3l$	$+L$	$+0{\cdot}8009514$
$+4D$		$-5l$		$-1{\cdot}2569452$
	$+2f$	$-l$		$+1{\cdot}0164958$
	$+2f$	$-l$	$+L$	$+1{\cdot}0912964$
	$+2f$	$-l$	$-L$	$+0{\cdot}9416952$
	$+2f$	$-3l$		$-0{\cdot}9666002$
	$+2f$	$-3l$	$+L$	$-0{\cdot}8917996$
	$+2f$	$-3l$	$-L$	$-1{\cdot}0414008$
		$+l$		$+0{\cdot}9915480$
		$+l$	$+L$	$+1{\cdot}0663486$
		$+l$	$+2L$	$+1{\cdot}1411492$
		$+l$	$+3L$	$+1{\cdot}2159498$
		$+l$	$-L$	$+0{\cdot}9167474$
		$+l$	$-2L$	$+0{\cdot}8419468$
		$+l$	$-3L$	$+0{\cdot}7671462$

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The following term will require special consideration :—

+D	+L	+0·9999993

This term depends on the angular distance of the Moon from the Solar perigee.

(B). The following Movements apply to inequalities of angular motion normal to the plane of the Ecliptic :—

3. *Inequalities of Long Period.*

Arguments.				Movements.
+ D	+ *f*	−2 *l*		−0·0538754
+ D	− *f*			−0·0788232
+ D	− *f*		+ L	−0·0040226
+ D	− *f*		− L	−0·1536238
+2D	+ *f*	−3 *l*		−0·1202247
+2D	+ *f*	−3 *l*	− L	−0·1950253
+2D	− *f*	− *l*		−0·1451725
+2D	− *f*	− *l*	+ L	−0·0703719
+2D	− *f*	− *l*	− L	−0·2199731
+2D	− *f*	− *l*	+2L	+0·0044287
+2D	−3*f*	+ *l*		−0·1701203
+2D	−3*f*	+ *l*	− L	−0·2449209
+3D	−3*f*			−0·2364696
	+ *f*	− *l*		+0·0124739
	+ *f*	− *l*	+ L	+0·0872745
	+ *f*	− *l*	+2L	+0·1620751
	+ *f*	− *l*	− L	−0·0623267
	+ *f*	− *l*	−2L	−0·1371273
	+3*f*	−3 *l*		+0·0374217

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4. *Inequalities whose Periods differ little from the Time of the Moon's Revolution.*

Arguments.				Movements.
+ D	+ f	− l		+0·9376726
+ D	+ f	− l	+ L	+1·0124732
+ D	− f	+ l		+0·9127248
+ D	− f	+ l	+ L	+0·9875254
+ D	− f	− l		−1·0703712
+ D	− f	− l	+ L	−0·9955706
+2D	+ f	−2l		+0·8713233
+2D	+ f	−2l	+ L	+0·9461239
+2D	+ f	−4l		−1·1117727
+2D	− f			+0·8463755
+2D	− f	−2l		−1·1367205
+2D	− f	−2l	+ L	−1·0619199
+2D	− f		+ L	+0·9211761
+2D	− f		+2L	+0·9959767
+2D	− f		− L	+0·7715749
+2D	−3f			−1·1616683
+2D	−3f		+ L	−1·0868677
	+ f			+1·0040219
	+ f	−2l		−0·9790741
	+ f	−2l	+ L	−0·9042735
	+ f	−2l	− L	−1·0538747
	+ f		+ L	+1·0788225
	+ f		− L	+0·9292213
	+ f		+2L	+1·1536231
	+ f		−2L	−0·8544207
	+3f	−2l		+1·0289697

For nearly the whole of these terms it will be necessary to carry the developments of series to one decimal place further than that which I contemplate as sufficient for final results, and, for some of them, to two decimal places further: that is, to 10^{-8} and 10^{-9} respectively, instead of 10^{-7}. This will be thus explained. For such developments as that of $\left(\frac{r}{a}\cos\overline{\mathrm{l}}\right)^2 \times \frac{dv}{dt}$, in the equation of areas, the series for $\left(\frac{r}{a}\cos\overline{\mathrm{l}}\right)^2$ must be expressed in its usual form, with coefficients to 10^{-7}; and the series for v must be expressed with the same exactness. Suppose now that one term in their product is A . sin $\overline{\mathrm{D} - 2f + l}$

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where A is expressed to 10^{-7}. The corresponding term in $\frac{dv}{dt}$ is $- \mathrm{A} \times 0·091297 . \cos |\overline{\mathrm{D} - 2f + l}|$, which is nearly $= \frac{-\mathrm{A}}{10} . \cos |\overline{\mathrm{D} - 2f + l}|$; and, if we do not express the new coefficient $\frac{-\mathrm{A}}{10}$ to 10^{-8}, we lose a part of the valid result of the application of this term, upon which the valid conclusion as to the correction of the coefficient A (by the process of Symbolical Variations) will hereafter depend. Therefore, the whole process for forming the term in $\left(\frac{r}{a} \cos |\overline{1}|\right)^2 \times \frac{dv}{dt}$ must be so conducted that the term whose argument is $\mathrm{D} - 2f + l$ shall be correct to 10^{-8}. And subsequently, when we form $\frac{d}{dt}\left\{\left(r \cos |\overline{1}|\right)^2 \times \frac{dv}{dt}\right\}$, the coefficient will again be multiplied by $-0·0912971$; and the product, to secure its legitimate validity, must be worked out correctly to 10^{-9}. In like manner, for the term $\mathrm{B} . \sin |\overline{f - \iota}|$, the first operation must be carried to 10^{-9} and the second to 10^{-11}.

For the arguments whose movements are nearly ± 1 the reasoning is somewhat different; but it leads to a conclusion almost similar, namely, that the extension of decimal places in the result must depend (in a similar manner) on the difference between the argument and the number ± 1. The small factor, however, is not here repeated.

1876, *January* 19.

Observations of Occultations of Stars by the Moon, and of Phenomena of Jupiter's Satellites, made at the Royal Observatory, Greenwich, in the year 1875.

(*Communicated by the Astronomer Royal.*)

Occultations of Stars by the Moon.

Day of Obs.	Phenomenon.	Telescope.*	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1875.						
May 12	Disapp. of 37 Leonis	E. Eq.	140	Dark	11 7 31·9	L
Oct. 16	,, ζ Arietis	Altaz.	100	Bright	9 20 16·6	J
,,	Reapp. of ζ Arietis	E. Eq.	140	Dark	10 14 37·4	C
Nov. 21	Disapp. of β Virginis	Altaz.	100	Bright	14 49 20·8	L
,,	Reapp. of β Virginis	,,	,,	Dark	15 54 30·2	L

* The clear aperture of the object-glass of the S.E. Equatoreal is $12\frac{3}{4}$ inches, of the East Equatoreal 6·7 inches, of the North Equatoreal 4·1 inches, and of the Altazimuth $3\frac{3}{4}$ inches.